

CLAIMS

1. A method of encoding over a Galois field  $\mathbf{F}_q$ , where  $q$  is an integer  
 5 greater than 2 and equal to a power of a prime number, in which a word  
 $\underline{v}' = (v'_0, v'_1, \dots, v'_{n'-1})$  is calculated, where  $n' \geq 3$ , belonging to an "MDS" linear  
 cyclic code  $C'$  of dimension  $(n'-m)$ , where  $1 \leq m \leq n'-2$ , on the basis of a word  
 $\underline{a} = (a_0, a_1, \dots, a_{n-m-1})$  of information symbols, where  $m < n < n'$ , characterized in  
 that a set  $\underline{s} = (s_0, s_1, \dots, s_{n-1})$  of strictly increasing integers, with  $s_0 \geq 0$  and  
 10  $s_{n-1} \leq n'-1$ , having been predetermined, it comprises the following steps:

a) forming the polynomial  $a(x) \equiv \sum_{i=m}^{n-1} a_{i-m} x^{s_i}$ ,

b) calculating the remainder  $r(x)$  of the Euclidean division of  $a(x)$  by

the polynomial  $g(x) \equiv \sum_{p=0}^m g_p x^p$  generating said code  $C'$ ,

c) calculating the polynomial  $\underline{v}^*(x) = a(x) - r(x) \equiv \sum_{i=0}^{n'-1} v^*_{i-1} x^i$ ,

15 corresponding to the word  $\underline{v}^* = (v^*_{0-1}, v^*_{1-1}, \dots, v^*_{n'-1})$ , and

d) taking  $\underline{v}' = \underline{v}^*$  if  $s_{m-1} = m-1$ ; otherwise obtaining said word  $\underline{v}'$  by  
 taking:

$$\underline{v}' = \underline{v}^* + \sum_{j=0}^{s_m-m-1} f_j \underline{\Gamma}^j, \quad (1)$$

in which the words  $\underline{\Gamma}^j$  of length  $n'$  are defined by:  $\Gamma^j_i = g_{i-j}$  for  $j \leq i \leq j+m$ ,

20 and  $\Gamma^j_i = 0$  otherwise, and in which the elements  $f_j$  of  $\mathbf{F}_q$  are calculated by  
 means of the equations (1) in which, for the  $(s_m - m)$  values of  $i < s_m$  not  
 belonging to the set  $\underline{s}$ , each component  $v'_i$  is taken equal to a respective  
 predetermined constant.

2. An encoding method according to claim 1, characterized in that it comprises an additional step consisting of deleting said components of predetermined value from  $\underline{v}'$ , so as to form a word  $\underline{v}$  of length  $n$ .

3. A method of encoding over a Galois field  $\mathbf{F}_q$ , where  $q$  is an integer

5 greater than 2 and equal to a power of a prime number, in which a word  $\underline{v}' = (v'_0, v'_1, \dots, v'_{n'-1})$  is calculated, where  $n' \geq 3$ , belonging to an "MDS" linear cyclic code  $C'$  of dimension  $(n'-m)$ , where  $1 \leq m \leq n'-2$ , on the basis of a word  $\underline{a} = (a_0, a_1, \dots, a_{n-m-1})$  of information symbols, where  $m < n < n'$ , characterized in that a set  $\underline{s} = (s_0, s_1, \dots, s_{n-1})$  of strictly increasing integers, with  $s_0 \geq 0$  and

10  $s_{n-1} \leq n'-1$ , and a non-singular diagonal matrix  $B$  of dimension  $n$  having been predetermined, it comprises the following steps:

- constructing, on the basis of the information symbols, the polynomial

$$a^B(x) \equiv \sum_{i=m}^{n-1} a_{i-m} \beta_{s_i}^{-1} x^{s_i},$$

where  $\beta_i$  is the element of  $B$  in position  $(i,i)$ ,

15 - implementing steps b), c) and d) of the method according to claim 1, by replacing  $a(x)$  with  $a^B(x)$ , which gives a word  $\underline{v}'^B$ ,

- deleting the components of predetermined value from  $\underline{v}'^B$ , which gives a word  $\underline{v}^B = (v^B_0, v^B_1, \dots, v^B_{n-1})$ , and

- calculating the word  $\underline{v} = (v_0, v_1, \dots, v_{n-1})$  defined by:  $v_i = v^B_i \beta_i$  for all  $i$

20 from 0 to  $(n-1)$ .

4. An encoding method according to any one of the preceding claims, characterized in that  $n'$  is equal to  $(q-1)$  or is a divisor of  $(q-1)$ , and in that

$$g(x) = \prod_{i=1}^m (x - \alpha^i), \text{ where } \alpha \text{ is an element of } \mathbf{F}_q \text{ satisfying } \alpha^{n'} = 1.$$

5. A method of encoding for algebraic geometric codes, comprising

25 at least one step in which codewords belonging to a shortened "MDS" linear cyclic code are calculated, characterized in that said calculation is performed by means of an encoding method according to any one of the preceding claims.

6. A device (102) for encoding over a Galois field  $\mathbb{F}_q$ , where  $q$  is an integer greater than 2 and equal to a power of a prime number, in which a word  $\underline{v}' = (v'_0, v'_1, \dots, v'_{n'-1})$  is calculated, where  $n' \geq 3$ , belonging to an "MDS" linear cyclic code  $C'$  of dimension  $(n'-m)$ , where  $1 \leq m \leq n'-2$ , on the basis of a word 5  $\underline{a} = (a_0, a_1, \dots, a_{n-m-1})$  of information symbols, where  $m < n < n'$ , characterized in that, a set  $\underline{s} = (s_0, s_1, \dots, s_{n-1})$  of strictly increasing integers, with  $s_0 \geq 0$  and  $s_{n-1} \leq n'-1$ , having been predetermined, it is adapted to:

a) form the polynomial  $a(x) = \sum_{i=m}^{n-1} a_{i-m} x^{s_i}$  ,  
 b) calculate the remainder  $r(x)$  of the Euclidean division of  $a(x)$  by the  
 10 polynomial  $g(x) = \sum_{p=0}^m g_p x^p$  generating said code  $C'$ ,

c) calculate the polynomial  $\underline{v}^*(x) = a(x) - r(x) = \sum_{i=0}^{n'-1} v^*_{i-1} x^i$ ,  
 corresponding to the word  $\underline{v}^* = (v^*_{0}, v^*_{1}, \dots, v^*_{n'-1})$ , and

d) take  $\underline{v}' = \underline{v}^*$  if  $s_{m-1} = m-1$ ; otherwise to obtain said word  $\underline{v}'$  by taking:

$$15 \quad \underline{v}' = \underline{v}^* + \sum_{j=0}^{s_m-m-1} f_j \underline{\Gamma}^j , \quad (1)$$

in which the words  $\underline{\Gamma}^j$  of length  $n'$  are defined by:  $\Gamma^j_i = g_{i-j}$  for  $j \leq i \leq j+m$ , and  $\Gamma^j_i = 0$  otherwise, and in which the elements  $f_j$  of  $\mathbb{F}_q$  are calculated by means of the equations (1) in which, for the  $(s_m - m)$  values of  $i < s_m$  not belonging to the set  $\underline{s}$ , each component  $v'_i$  is taken equal to a respective 20 predetermined constant.

7. An encoding device according to claim 6, characterized in that  $n'$  is equal to  $(q-1)$  or is a divisor of  $(q-1)$ , and in that  $g(x) = \prod_{i=1}^m (x - \alpha^i)$ , where  $\alpha$  is an element of  $\mathbb{F}_q$  satisfying  $\alpha^{n'} = 1$ .

5 8. An apparatus (48) for processing data comprising a source of information symbols (100), characterized in that it further comprises:

- a storage unit (101) adapted to accumulate said symbols so as to form codewords a each containing a predetermined number  $k$  of symbols,
- an encoding device according to claim 6 or claim 7, and
- a transmitter (103) adapted to transmit the words y' resulting from the 10 encoding of said information symbols.

9. An apparatus (48) for processing data comprising a source of information symbols (100), characterized in that it further comprises:

- a storage unit (101) adapted to accumulate said symbols so as to form codewords a each containing a predetermined number  $k$  of symbols,
- 15 - an encoding device according to claim 6 or claim 7,
- a shortening unit (20) adapted to delete said components of predetermined value from y', so as to form a word y of length  $n$ , and
- a transmitter (103) adapted to transmit the words y resulting from the encoding of said information symbols.

20 10. A non-removable data storage means, characterized in that it comprises computer program code instructions for the execution of the steps of an encoding method according to any one of claims 1 to 5.

25 11. A partially or wholly removable data storage means, characterized in that it comprises computer program code instructions for the execution of the steps of an encoding method according to any one of claims 1 to 5.

30 12. A computer program, characterized in that it contains instructions such that, when said program controls a programmable data processing device, said instructions lead to said data processing device implementing an encoding method according to any one of claims 1 to 5.

13. A method of encoding information by means of a given code shortened in at least one predetermined position, comprising the steps of:

- dividing a first polynomial representing information symbols by a generator polynomial so as to obtain a remainder polynomial;

5 - calculating a second polynomial by subtracting the remainder polynomial from the first polynomial;

- generating a pre-encoded word belonging to the given code from a sequence of coefficients of the second polynomial, so that the component of the pre-encoded word in the at least one predetermined position has a respective

10 predetermined value; and

- generating an encoded word by removing from the pre-encoded word the component in the at least one predetermined position.

14. A method of encoding for algebraic geometric codes, comprising at least one step in which codewords belonging to a shortened code are

15 calculated, characterized in that the calculation of the codewords belonging to the shortened code is performed by using an encoding method according to Claim 13.

15. A device for encoding information by means of a given code shortened in at least one predetermined position, comprising:

20 - means for dividing a first polynomial representing information symbols by a generator polynomial so as to obtain a remainder polynomial;

- means for calculating a second polynomial by subtracting the remainder polynomial from the first polynomial;

25 - means for generating a pre-encoded word belonging to the given code from a sequence of coefficients of the second polynomial, so that the component of the pre-encoded word in the at least one predetermined position has a respective predetermined value; and

- means for generating an encoded word by removing from the pre-encoded word the component in the at least one predetermined position.

30 16. Information storage medium which can be read by a computer or a microprocessor storing instructions of a computer program for implementing a coding method according to any one of Claims 13 and 14.

17. Information storage medium according to Claim 16, characterized in that said storage medium is partially or totally removable.

18. Computer program product comprising sequences of instructions for implementing a coding method according to any of Claims 13 and 14.